

Covering logics via Possible-Translations

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Abstract

We present a general study of a new formalism of decomposition of logics, the *Possible-Translations Coverings* (in short *PTC*'s) which constitute a formal version of Possible-Translations Semantics, introduced by W. Carnielli in 1990. We show how the adoption of a more general notion of propositional signatures morphism allows us to define a category \mathbf{Sig}_ω , in which the connectives, when translated from a signature to another one, enjoy of great flexibility. Essentially, \mathbf{Sig}_ω -morphisms will be multifunctions instead of functions. Using morphisms in \mathbf{Sig}_ω we define the notion of a *PTC* for a logic \mathcal{L} . We analyze some properties of *PTC*'s and give concrete examples of some of the above mentioned constructions. We conclude indicating the first steps to be followed in order to characterize *PTC*'s as a universal construction in an adequately defined category of logics.

1 Preliminaries

We list here facts and definitions that will be used throughout this paper.

Definition 1. *A propositional signature is any family of sets $C = \{C_k\}_{k \in \mathbb{N}}$ such that $C_i \cap C_j = \emptyset$, if $i \neq j$. The elements of the set C_k are called k -ary connectives. In particular, the elements of C_0 are called constants. We define $|C| \stackrel{def}{=} \bigcup_{n \in \mathbb{N}} C_n$.*

We assume that $\mathcal{V} = \{p_n \mid n \in \mathbb{N}\}$ is a set of propositional variables.

Definition 2. *Let C be a signature. The language generated by C is the least set $L(C)$ that satisfies the following properties:*

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- $\mathcal{V} \subseteq L(C)$;
- if $n \in \mathbb{N}$, $c \in C_n$ and $\varphi_1, \dots, \varphi_n \in L(C)$, then $c(\varphi_1, \dots, \varphi_n) \in L(C)$.

The elements of $L(C)$ are called formulas.

Definition 3. A substitution over C is a map $\sigma : \mathcal{V} \rightarrow L(C)$. Each substitution admits a unique extension $\hat{\sigma} : L(C) \rightarrow L(C)$ such that:

1. $\hat{\sigma}(p) = \sigma(p)$, if $p \in \mathcal{V}$;
2. $\hat{\sigma}(c) = c$, if $c \in C_0$;
3. $\hat{\sigma}(c(\varphi_1, \dots, \varphi_n)) = c(\hat{\sigma}(\varphi_1), \dots, \hat{\sigma}(\varphi_n))$, if $c \in C_n$ and $\varphi_1, \dots, \varphi_n \in L(C)$.

In the sequel, $L(C)_n$ will denote the set of formulas $\varphi \in L(C)$ such that the set of propositional variables occurring in φ is contained in $\{p_1, \dots, p_n\}$. If $\varphi \in L(C)_n$ we can write $\varphi(p_1, \dots, p_n)$. If σ is a substitution over C such that $\sigma(p_i) = \varphi_i$ for $1 \leq i \leq n$, then $\varphi(\varphi_1, \dots, \varphi_n)$ denotes $\sigma(\varphi)$. The proof of the following lemma can be found in [Bueno-Soler *et al.*, 2006].

Lemma 4. Let $\varphi(p_1, \dots, p_n)$ be a formula, and $\sigma, \sigma' : \mathcal{V} \rightarrow L(C)$ be substitutions such that $\sigma(p_i) = \varphi_i$, for $1 \leq i \leq n$. Then $\hat{\sigma}'(\varphi(\varphi_1, \dots, \varphi_n)) = \varphi(\hat{\sigma}'(\varphi_1), \dots, \hat{\sigma}'(\varphi_n))$.

Definition 5. A (propositional) logic is a pair $\mathcal{L} = \langle C, \vdash_{\mathcal{L}} \rangle$, where C is a signature and $\vdash_{\mathcal{L}}$ is a subset of $\wp(L(C)) \times L(C)$ satisfying the following properties, for every $\Gamma \cup \Theta \cup \{\varphi\} \subseteq L(C)$:

- If $\varphi \in \Gamma$ then $\Gamma \vdash_{\mathcal{L}} \varphi$ (Extensivity);
- If $\Gamma \vdash_{\mathcal{L}} \varphi$ and $\Theta \vdash_{\mathcal{L}} \psi$ for all $\psi \in \Gamma$ then $\Theta \vdash_{\mathcal{L}} \varphi$ (Transitivity);
- If $\Gamma \vdash_{\mathcal{L}} \varphi$ then $\Delta \vdash_{\mathcal{L}} \varphi$ for some finite set $\Delta \subseteq \Gamma$ (Finitariness);
- If $\Gamma \vdash_{\mathcal{L}} \varphi$ then $\hat{\sigma}(\Gamma) \vdash_{\mathcal{L}} \hat{\sigma}(\varphi)$ for every substitution σ (Structurality).

The relation $\vdash_{\mathcal{L}}$ is called the consequence relation of \mathcal{L} .

Definition 6. Let $\mathcal{L} = \langle C, \vdash_{\mathcal{L}} \rangle$ and $\mathcal{L}' = \langle C', \vdash_{\mathcal{L}'} \rangle$ be two logics. A logic translation from \mathcal{L} to \mathcal{L}' is a map $\lambda : L(C) \rightarrow L(C')$ which preserves derivability, that is: if $\Gamma \vdash_{\mathcal{L}} \varphi$ then $\lambda(\Gamma) \vdash_{\mathcal{L}'} \lambda(\varphi)$ for every $\Gamma \cup \{\varphi\} \subseteq L(C)$.

Definition 7. Let X and Y be non-empty sets. A finite multifunction $f : X \rightarrow Y$ is a point-to-set correspondence from X to Y , that is, for each $x \in X$, $f(x)$ is a non-empty finite subset of Y .

In what follows, the symbol $\wp_f X^+$ stands for the set of non-empty finite subsets of X . Thus, a finite multifunction $f : X \rightarrow Y$ is nothing else than a function $f : X \rightarrow \wp_f X^+$. Since we will only be concerned with finite multifunctions, we will omit the adjective finite. Now we define a composition operation $\bar{\circ}$ for multifunctions.

Definition 8. *Given multifunctions $f : X \rightarrow Y$ and $g : Y \rightarrow Z$, we define the composition $g\bar{\circ}f$ as the multifunction $g\bar{\circ}f : X \rightarrow Z$ by $g\bar{\circ}f(x) \stackrel{def}{=} \bigcup_{y \in f(x)} g(y)$.*

The next result will be used later to establish some relevant results.

Lemma 9. *The operation $\bar{\circ}$ is associative.*

If $f : X \rightarrow Y$ is a multifunction and $A \subseteq X$, we use - when convenient - the symbol $f(A)$ to denote the set $\bigcup_{a \in A} f(a)$.

2 The category Sig_ω

Consider the class *Sig* of propositional signatures. We introduce the following notion: a *signature morphism* $f : C^1 \rightarrow C^2$ from C^1 to C^2 is a multifunction $f : |C^1| \rightarrow L(C^2)$. The identity id_{C^1} is defined as the multifunction $id_{C^1} : |C^1| \rightarrow L(C^1)$ such that $id_{C^1}(c) = \{c(p_1, \dots, p_n)\}$, for $c \in C_n^1$. Intuitively, we identify a n -ary connective with a finite set of formulas (of the language generated by the target signature) in which occur at most the propositional variables p_1, \dots, p_n . Recalling the notation introduced in Section 1, a n -ary connective can be seen as an element of $\wp_f(L(C^2)_n)^+$.

Example 10. *Consider the signatures $C^1 = \{\neg_1, \Rightarrow\}$, $C^2 = \{\neg_2, \vee, \wedge\}$. The map f defined as follows is a signature morphism from C^1 in C^2 :*

- $f(\neg_1) = \{\neg_2 p_1\}$;
- $f(\Rightarrow) = \{\neg_2 p_1 \vee p_2, \neg_2(p_1 \wedge \neg_2 p_2)\}$.

Definition 11. *Let $f : C^1 \rightarrow C^2$ be a signature morphism. We can define its extension \hat{f} as the multifunction $\hat{f} : L(C^1) \rightarrow L(C^2)$ such that*

1. $\hat{f}(p) = \{p\}$, if $p \in \mathcal{V}$;
2. $\hat{f}(c) = f(c)$, if $c \in C_0^1$;
3. $\hat{f}(c(\varphi_1, \dots, \varphi_n)) = \{\varphi(\psi_1, \dots, \psi_n) \mid \varphi \in f(c), \psi_i \in \hat{f}(\varphi_i)\}$, if $c \in C_n^1$ and $\varphi_1, \dots, \varphi_n \in L(C^1)$.

It is easily provable - by induction on the complexity of the formula - that such an extension is unique.

Example 12. *Returning to Example 10, we have for each $\varphi, \psi \in L(C^1)$,*

- $\hat{f}(\neg\varphi) = \{\neg_2\gamma \mid \gamma \in \hat{f}(\varphi)\}$
- $\hat{f}(\varphi \Rightarrow \psi) = \{\neg_2\gamma \vee \delta, \neg_2(\gamma \wedge \neg_2\delta) \mid \gamma \in \hat{f}(\varphi), \delta \in \hat{f}(\psi)\}$.

Now, consider the formula $\varphi = (p_1 \Rightarrow (p_2 \Rightarrow p_3))$. We have $\hat{f}(\neg_1\varphi) = \hat{f}(\neg_1(p_1 \Rightarrow (p_2 \Rightarrow p_3))) = \{\neg_2\gamma \mid \gamma \in \hat{f}(p_1 \Rightarrow (p_2 \Rightarrow p_3))\}$. Also, $\hat{f}(p_1 \Rightarrow (p_2 \Rightarrow p_3)) = \{\neg_2p_1 \vee \psi, \neg_2(p_1 \wedge \neg_2\psi) \mid \psi \in \hat{f}(p_2 \Rightarrow p_3)\} = \{\neg_2p_1 \vee (\neg_2p_2 \vee p_3), \neg_2p_1 \vee \neg_2(p_2 \wedge \neg_2p_3), \neg_2(p_1 \wedge \neg_2(\neg_2p_2 \vee p_3)), \neg_2(p_1 \wedge \neg_2\neg_2(p_1 \wedge \neg_2p_3))\}$. We obtain $\hat{f}(\neg_1\varphi) = \{\neg_2\gamma \mid \gamma \in \hat{f}(\varphi)\} = \hat{f}(\neg_1(p_1 \Rightarrow (p_2 \Rightarrow p_3))) = \{\neg_2(\neg_2p_1 \vee (\neg_2p_2 \vee p_3)), \neg_2(\neg_2p_1 \vee \neg_2(p_2 \wedge \neg_2p_3)), \neg_2\neg_2(p_1 \wedge \neg_2(\neg_2p_2 \vee p_3)), \neg_2\neg_2(p_1 \wedge \neg_2\neg_2(p_1 \wedge \neg_2p_3))\}$.

The example above suggests that we must look for a more convenient notation to express the facts about the objects we study. It justifies the following conventions.

Notation 13. *Let $\Gamma, \Gamma_1, \dots, \Gamma_n$ be non-empty subsets of $L(C^1)$. We may use $\Gamma(\Gamma_1, \dots, \Gamma_n)$ to denote the set $\{\gamma(\gamma_1, \dots, \gamma_n) \mid \gamma \in \Gamma, \gamma_i \in \Gamma_i\}$. If c is a n -ary connective, the expression $c(\Gamma_1, \dots, \Gamma_n)$ will be used to denote the set $\{c(\gamma_1, \dots, \gamma_n) \mid \gamma_i \in \Gamma_i\}$.*

Of course we have a notion of composition to our morphisms, and we give it below.

Definition 14. *Let $f : C^1 \rightarrow C^2$ and $g : C^2 \rightarrow C^3$ be signature morphisms. The composition of g and f is the signature morphism $g \cdot f : C^1 \rightarrow C^3$ defined by $(g \cdot f)(c) = \cup_{\psi \in f(c)} \hat{g}(\psi)$.*

It is easy to see, by the very definition of the composition, that the identity works as expected.

We can ask if the composition \cdot is associative. The answer is affirmative, and to show it we will need some technical results, which we display in the following lemmas.

Lemma 15. *Let $f : C^1 \rightarrow C^2$, $g : C^2 \rightarrow C^3$ be morphisms and $c \in C_n^1$. Then $(g \cdot f)(c) = (\widehat{g \circ f})(c(p_1, \dots, p_n))$.*

Lemma 16. *Let $f : C^1 \rightarrow C^2$, $g : C^2 \rightarrow C^3$ be morphisms, and $\varphi \in L(C^1)$. Then $\hat{f}(\varphi(\alpha_1, \dots, \alpha_n)) = \hat{f}(\varphi)(\hat{f}(\alpha_1), \dots, \hat{f}(\alpha_n))$.*

Lemma 17. *Let $f : C^1 \rightarrow C^2$, $g : C^2 \rightarrow C^3$ be morphisms. Then $\widehat{g \cdot f} = \widehat{g} \circ \widehat{f}$.*

Proposition 18. *The composition \cdot is associative.*

Proof. Consider the following configuration $C^1 \xrightarrow{f} C^2 \xrightarrow{g} C^3 \xrightarrow{h} C^4$ in *Sig*. We have $[h \cdot (g \cdot f)] \stackrel{cf. 15}{=} [\widehat{h \circ (g \cdot f)}] \stackrel{cf. 17}{=} [\widehat{h \circ (\widehat{g \cdot f})}] \stackrel{cf. 9}{=} [(\widehat{h \circ g}) \circ \widehat{f}] \stackrel{cf. 17}{=} [(\widehat{h \cdot g}) \circ f] \stackrel{cf. 15}{=} [(h \cdot g) \cdot f] \quad \square$

All together gives that the class *Sig* with the morphisms we introduced is a category. From now on, we will denote this category by \mathbf{Sig}_ω .

We want now investigate the existence of products in \mathbf{Sig}_ω . For this purpose, consider a non-empty set I and a collection $\mathcal{F} = \{C^i\}_{i \in I}$ of signatures. We have a natural candidate to the product $(\mathcal{C}^{\mathcal{F}}, \{\pi_i\}_{i \in I})$:

$$\mathcal{C}_k^{\mathcal{F}} = \{(\Gamma_i)_{i \in I} \mid \Gamma_i \in \wp_f(L(C^i)_k)\} \text{ for } k \in \mathbb{N};$$

$$\mathcal{C}_k^{\mathcal{F}} \ni (\Gamma_i)_{i \in I} \xrightarrow{\pi_j} \Gamma_j \in \wp_f(L(C^j)_k) \text{ for } j \in I.$$

Unfortunately, this candidate is not a product, because it fails in being unique, that is, this pair has the universal property of products up to uniqueness. This is what is called a weak product.

Proposition 19. *The pair $\langle \mathcal{C}^{\mathcal{F}}, \{\pi_i\}_{i \in I} \rangle$ defined above is a weak product of \mathcal{F} in \mathbf{Sig}_ω .¹*

Proof. It is easy to see that each π_i is a \mathbf{Sig}_ω -morphism from $\mathcal{C}^{\mathcal{F}}$ to C^i . Consider now a signature C' and morphisms $C' \xrightarrow{f_i} C^i$ for each $i \in I$. Define $f : C' \rightarrow \mathcal{C}^{\mathcal{F}}$ by $C'_n \ni c \xrightarrow{f} \{(f_i(c))_{i \in I}(p_1, \dots, p_n)\}$. Then $(\pi_i \cdot f)(c) = \widehat{\pi}_i(f(c)) = f_i(c)$. \square

Although we don't have "full" products, this weak product is enough to our purposes.

Example 20. *Let us consider a weak product C in \mathbf{Sig}_ω of the signatures C^1 and C^2 from **Example 10**. These are some of the connectives of the weak product signature:*

- $(\{\neg_1 p_1\}, \{\neg_2 p_1\})$ is a unary connective in C .

¹Here we need to point out rectification to Proposition 2.14 of [Bueno-Soler *et al.*, 2006], which is a particular case of our proposition (since any function is a multifunction). Even in the case in which the morphisms are functions, in general one can't assure the existence of a product, but only of a weak product. We thank Hugo Mariano for this observation.

- $(\{p_1 \Rightarrow p_2\}, \{p_1 \wedge p_2\}) \in C_2$ (that is, it is a binary connective in C).
- $(\{\neg_1 p_2 \Rightarrow \neg_1 p_1\}, \{\neg_2 p_1 \vee (\neg_2 p_1 \wedge p_2), p_1 \wedge p_2, p_1 \vee \neg_2 p_2, p_1 \vee \neg_2 (p_2 \vee \neg_2 p_2)\})$ is another binary connective in C .

Note that, in the present situation, $C_i \neq \emptyset$ if $i \geq 3$.

We end this section with a remark about substitutions: they can be composed with signature morphisms. Firstly, observe that, given a substitution $\sigma : \mathcal{V} \rightarrow L(C)$, it gives rise to a multifunctional substitution $\bar{\sigma} : \mathcal{V} \rightarrow L(C)$ defined by $\bar{\sigma}(p) = \{\sigma(p)\}$. Clearly $\bar{\sigma}$ can be extended to a unique multifunction $\hat{\sigma} : L(C) \rightarrow L(C)$ as expected: $\hat{\sigma}(\varphi) = \{\bar{\sigma}(\varphi)\}$. Since this association is quite natural, we may identify the substitutions σ and $\bar{\sigma}$, and consequently we also may identify $\hat{\sigma}$ and $\widehat{\bar{\sigma}}$. Hereby, we can consider the composition of signature morphisms and substitutions (recall Definition 8). Thus, if $f : C \rightarrow C'$ is a signature morphism and σ and σ' are substitutions over C and C' , respectively, then $f \circ \sigma$ and $\hat{\sigma}' \circ f$ will stand for $f \circ \bar{\sigma}$ and $\widehat{\bar{\sigma}'} \circ f$, respectively.

3 Possible-translations coverings

In this section we introduce the central notion of possible-translations coverings. Previous to this, we need some additional definitions and results.

Definition 21. Let $f : C^1 \rightarrow C^2$ be a signature morphism. An instance of f is a function $\lambda : L(C^1) \rightarrow L(C^2)$ defined inductively such that:

- $\lambda(p) = p$, if $p \in \mathcal{V}$.
- let $c \in C_n^1$ and $\varphi_1, \dots, \varphi_n \in L(C^1)$ be such that $\lambda(\varphi_i)$ was already defined, for $1 \leq i \leq n$. Then $\lambda(c(\varphi_1, \dots, \varphi_n)) = c(\lambda(\varphi_1), \dots, \lambda(\varphi_n))$, for some formula $\varphi \in f(c)$.

The set of instances of f will be denoted by $\text{Ins}(f)$

The proof of the next lemma is straightforward.

Lemma 22. Let $f : C^1 \rightarrow C^2$ and $g : C^2 \rightarrow C^3$ be morphisms, $\lambda \in \text{Ins}(f)$ and $\mu \in \text{Ins}(g)$. Then $\text{Ins}(id_{C^1}) = \{\widehat{id_{C^1}}\}$, and $\mu \circ \lambda \in \text{Ins}(g \cdot f)$.

With the definitions above introduced, we are now ready to introduce the envisaged notion of possible-translations coverings:

Definition 23. Let $\mathcal{L} = \langle C, \vdash \rangle$ be a logic. A covering of \mathcal{L} by possible-translations (in short, a PTC for \mathcal{L}) is a structure $P = \langle f, \Lambda, \{\mathcal{L}_\lambda\}_{\lambda \in \Lambda} \rangle$ such that:

- $f : C \rightarrow C^1$ is a signature morphism.
- Λ is a subset of $\text{Ins}(f)$.
- $\mathcal{L}_\lambda = \langle C^1, \vdash_\lambda \rangle$ is a logic for each $\lambda \in \Lambda$.
- $\lambda : \mathcal{L} \rightarrow \mathcal{L}_\lambda$ is a logic translation, for each $\lambda \in \Lambda$.

The elements of Λ are called possible-translations.

Any *PTC* defines a logic in a natural way:

Definition 24. Given a logic $\mathcal{L} = \langle C, \vdash \rangle$ let $P = \langle f, \Lambda, \{\mathcal{L}_\lambda\}_{\lambda \in \Lambda} \rangle$ be a *PTC* for \mathcal{L} . The logic associated to P is the pair $\mathcal{L}_P = \langle C, \vdash_P \rangle$ such that \vdash_P is a subset of $\varphi(L(C)) \times L(C)$ defined as follows:

$$\Gamma \vdash_P \varphi \text{ if, and only if, } \lambda(\Gamma) \vdash_\lambda \lambda(\varphi), \text{ for all } \lambda \in \Lambda$$

Using previous results it is possible to prove that \mathcal{L}_P is in fact a logic. Moreover, $\vdash \subseteq \vdash_P$, that is: $\Gamma \vdash \varphi$ implies that $\Gamma \vdash_P \varphi$, for every $\Gamma \cup \{\varphi\} \subseteq L(C)$. This suggest the following definition:

Definition 25. Let P be a *PTC* for $\mathcal{L} = \langle C, \vdash \rangle$. We say that P is adequate for \mathcal{L} if $\vdash = \vdash_P$, that is: $\Gamma \vdash \varphi$ if and only if $\Gamma \vdash_P \varphi$, for every $\Gamma \cup \{\varphi\} \subseteq L(C)$.

Thus, if P is a *PTC* adequate for \mathcal{L} then \mathcal{L} is decomposed into the factors \mathcal{L}_λ through the translations λ .

It is easy to see that *PTC's* generalize the notion of possible-translations semantics and, therefore, they generalize matrix semantics and non-deterministic semantics (cf. [Carnielli and Coniglio, 2005]). On the other hand, it is a particular case of possible-translations representations (see [Marcos, 2004]).

4 The category Log_ω

It is possible to characterize, in some specific cases, the notion of possible-translations semantics in categorial terms (cf. [Carnielli and Coniglio, 2005]). Thus, a *PTS* for \mathcal{L} is a conservative translation² from \mathcal{L} to a weak product of logics, where the weak product is computed in an appropriate category of logics. This suggest the possibility of obtaining a similar result for possible-translations coverings. The first step consist of defining an appropriate category of logics, and then prove that this category has weak products.

²A logic translation $f : \mathcal{L} \rightarrow \mathcal{L}'$ is said to be *conservative* if: $\Gamma \vdash_{\mathcal{L}} \varphi$ if and only if $\Gamma \vdash_{\mathcal{L}'} \varphi$

With this motivation, in this section we propose the category \mathbf{Log}_ω of logics, based on \mathbf{Sig}_ω , and prove that \mathbf{Log}_ω has weak products. The fundamental step is the definition of morphisms between logics.

Definition 26. Let $\mathcal{L}_i = \langle C^i, \vdash_i \rangle$ be logics ($i = 1, 2$). A morphism of logics $f : \mathcal{L}_1 \rightarrow \mathcal{L}_2$ is a signature morphism $f : C^1 \rightarrow C^2$ such that, for all $\Gamma \cup \{\varphi\} \subseteq L(C^1)$ satisfying $\Gamma \vdash_1 \varphi$, there exists an instance λ (depending on Γ and φ) of f such that $\lambda(\Gamma) \vdash_2 \lambda(\varphi)$.

Using the definition above, together with the previous lemma, we get the category \mathbf{Log}_ω of logics.

Let us see that the category \mathbf{Log}_ω has weak products. Consider a non-empty small family $\mathcal{F} = \{\mathcal{L}_i\}_{i \in I}$ in \mathbf{Log}_ω , $\mathcal{L}_i = \langle C^i, \vdash_i \rangle$, for all $i \in I$. Let $\langle \mathcal{C}^\mathcal{F}, \{\pi_i\}_{i \in I} \rangle$ be a weak product of $\{C^i\}_{i \in I}$ in \mathbf{Sig}_ω . One hopes that the weak product - if it exists - must be constructed from a weak product in \mathbf{Sig}_ω . Specifically, we intend to define a logic $\prod \mathcal{L}_i = \langle \mathcal{C}^\mathcal{F}, \vdash \rangle$. In view of our definition of morphism in \mathbf{Log}_ω , the consequence relation \vdash of a weak product should be something like this:

$\Upsilon \vdash \Psi$ if, and only if, there is an instance λ_i of π_i such that $\lambda_i(\Upsilon) \vdash_i \lambda_i(\Psi)$,
for all $i \in I$.

However, it is not always the case that the object described above is a logic. So, we use the inferences $\Upsilon \vdash \Psi$ as above as an inferential basis for the consequence relation \vdash of $\prod \mathcal{L}_i$ (cf. [Łoś and Suszko, 1958]). That is, \vdash is the least consequence relation over the signature $\mathcal{C}^\mathcal{F}$ containing such inferences. This is our candidate for a weak product. Clearly, each π_i is a \mathbf{Log}_ω -morphism $\pi_i : \prod \mathcal{L}_i \rightarrow \mathcal{L}_i$. This proves the following:

Proposition 27. The pair $\langle \prod \mathcal{L}_i, \{\pi_i\}_{i \in I} \rangle$ defined above is in fact a weak product of \mathcal{F} in \mathbf{Log}_ω .

It is an interesting question to determine if a categorial characterization of possible-translations coverings is possible, by using the category \mathbf{Log}_ω as a basis.

5 Conclusions

In this paper was presented a wide notion of morphism signature by using multifunctions. This notion captures, in some sense, the concept of ‘flexible connective’, that is, a connective which can be interpreted in several (but controlled) ways. The proposed category of signatures was used to define the

concept of possible-translations coverings, which generalizes the notion of possible-translations semantics. Despite being a particular case of possible-translations representations, the restrictions imposed in *PTC*'s to the translations (say, to be a set of instances of a given signature morphism) suggests that certain logics are based on flexible (that is, varying) connectives. Thus, for such logics, an adequate *PTC* enlightens this features.

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